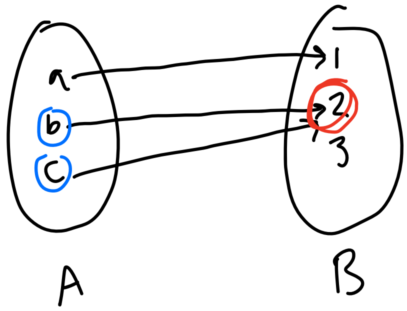
One to one

$f: A \to B$. if $y \in B$ and $x \in A$, $f(x) = y$ then x is $\boxed{a}$ preimage of y.

there could be more than one.



$g: A \to B$

this is a function: every element of the domain should have exactly one output.

but not one-to-one

A function is one-to-one if two input values never give the same output; or if no element of the co-domain has more than one pre-image.

let $f: A \to B$

$$\forall x, y \in A, \quad x \neq y \rightarrow f(x) \neq f(y)$$

for proofs, use contrapositive definition (equivalent definition)

$$\forall x, y \in A, \quad f(x) = f(y) \rightarrow x = y$$

similar to proof technique for antisymmetric.

if a function is one to one and onto, it is a bijection

$f: A \to B$ is a bijection

① $f^{-1}: B \to A$ exists and is also a bijection

② $|A| = |B|$

one-to-one proof: (concrete)

Let $f: \mathbb{Z} \to \mathbb{Z}$ and $f(x) = 2x+1$. Prove f is one to one.

Let $x, y \in \mathbb{Z}$. And suppose $f(x) = f(y)$. (hypothesis of contrapos. def'n of one-to-one).

Then, $2x+1 = 2y+1$. (using def'n of f).

$2x = 2y$

$x = y$ (conclusion of def'n one-to-one)

So f is one-to-one.

one-to-one proof (abstract with composition):

For sets A, B, C and for functions $f: A \to B$, $g: B \to C$,

if $\underbrace{f \text{ and } g \text{ are one-to-one}}_{\text{hyp.}}$, then $(g \circ f)$ is also one-to-one.

$g \circ f : \underset{\text{domain}}{A} \to \underset{\text{co-domain}}{C}$

Let A, B, C be sets and let $f: A \to B$, $g: B \to C$ be functions.

$\underbrace{\text{Suppose } f \text{ and } g \text{ are one to one.}}_{\text{hyp.}}$

Let $x, y \in A$, and suppose $g(\overset{a}{f(x)}) = g(\overset{b}{f(y)})$

Since g is one-to-one, it must be the case $f(x) = f(y)$.

Then, since f is also one-to-one, it must be case that $x = y$.

So $g \circ f$ is also one-to-one.